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# Light exotic nuclei studied with the parity-projected Hartree-Fock method

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**Abstract.** Calculation of the variation after parity projection is performed in 3D Cartesian mesh representation for ground and excited states of even-even magnesium isotopes. The full 3D angular momentum projection after the variation provides rotational spectra for deformed nuclei. The method takes account of a part of octupole and rotational correlations beyond the mean field. We show the positive-parity ground band and the lowest negative-parity excited band in each isotope. Properties of the ground-state bands are in a good agreement with experiments.

**PACS.** 21.60.Jz Hartree-Fock and random-phase approximations

## 1 Introduction

Among theories beyond the mean field, the variation after projection (VAP) method is one of the simplest ones. Yet, practical applications with realistic effective interactions, such as the Skyrme interaction, have not been fully investigated even for projection with respect to just the parity. The variation after the parity projection (VAPP) with a symmetry-violating intrinsic state leads to the ground state including some octupole correlations. In addition, one can obtain a negative-parity excited state which is either a collective or non-collective excitation. Recently, we propose an algorithm to calculate the VAPP with the Skyrme interaction on the three-dimensional (3D) Cartesian coordinates [1]. The calculations with the simple BKN interaction were reported before [2].

In the ordinary mean-field calculations, one finds self-consistent solutions with axial and reflection symmetries for most nuclei. However, in the VAPP calculations, the self-consistent solutions violate these symmetries. This symmetry violation in the intrinsic state is a consequence of the correlation beyond the mean field, which can be regarded as the octupole correlation. This is similar to cluster correlation in light systems. In ref. [1], we have shown that the cluster structure of $\alpha + {}^{16}$O appears in $^{20}$Ne and the three $\alpha$ structure in $^{12}$C, as a result of the VAPP using the Skyrme interaction. Simultaneously, we have obtained negative-parity excited bands with both well-developed cluster structures and shell-model-like particle-hole excitations in these nuclei. The angular momentum projection after the variation well reproduces experimental low-lying spectra and transition strengths.

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In this paper, we apply the same method to Mg isotopes. Among those isotopes, neutron-rich nuclei in the neighborhood of the shell closure $N = 20$ are of significant interest. The observed low excitation energy of $2^+$ state and the enhanced $B(E2; 0^+ \to 2^+)$ for $^{32}$Mg suggests an anomalous deformation and a quenching of the shell gap at $N = 20$ [3]. The deformation becomes more evident for $^{34}$Mg, in which the $2^+$ excitation energy is even lower and the $B(E2)$ is larger than those in $^{32}$Mg [4].

## 2 VAP calculation in the 3D coordinate space

In this section, we briefly summarize our method of VAP calculation. The details are given in refs. [1,2].

The method is based on minimization of the energy expectation value with respect to a parity projected Slater determinant, $|\Phi^{(\pm)}\rangle = \frac{1}{\sqrt{2}}(1 \pm \hat{P})|\Phi\rangle$, where $\hat{P}$ is the space inversion operator and $|\Phi\rangle = \det\{|\phi_i\rangle\}/\sqrt{N!}$ represents an intrinsic Slater determinant. The variation with respect to the single-particle orbitals $|\phi_i\rangle$ leads to

$$(h - \boldsymbol{\eta}\cdot\boldsymbol{r})\,|\phi_i\rangle \pm \langle\Phi|\hat{P}|\Phi\rangle\left\{h_P|\tilde{\phi}_i\rangle - \sum_j |\tilde{\phi}_j\rangle\langle\phi_j|h_P|\tilde{\phi}_i\rangle\right\} + (E^{(\pm)} - E)|\tilde{\phi}_i\rangle = \sum_j \epsilon_{ij}|\phi_j\rangle, \quad (1)$$

where $E = \langle\Phi|H|\Phi\rangle$ and $E^{(\pm)} = \langle\Phi^{(\pm)}|H|\Phi^{(\pm)}\rangle$. $h$ is the usual Hartree-Fock Hamiltonian. $h_P$ has the same structure as $h$, however, all the densities are replaced by the transition densities [2]. $|\tilde{\phi}_i\rangle$ is defined by $\sum_j \hat{P}|\phi_j\rangle(B^{-1})_{ji}$, with $B_{ij} = \langle\phi_i|\hat{P}|\phi_j\rangle$. $\boldsymbol{\eta}$ and $\epsilon_{ij}$ are the

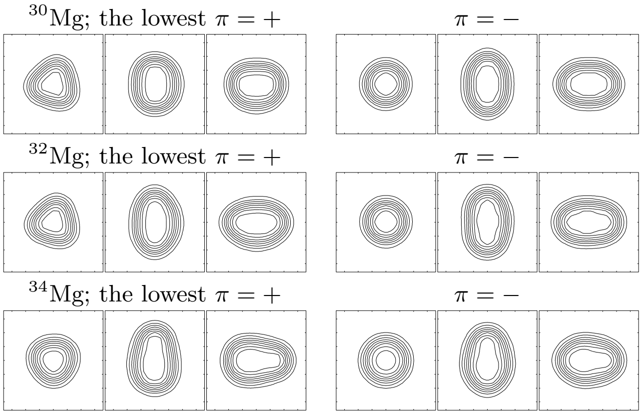


**Fig. 1.** Contour plots of intrinsic density in the $xy$-, $yz$-, and $zx$-planes for the lowest state in each parity sector.

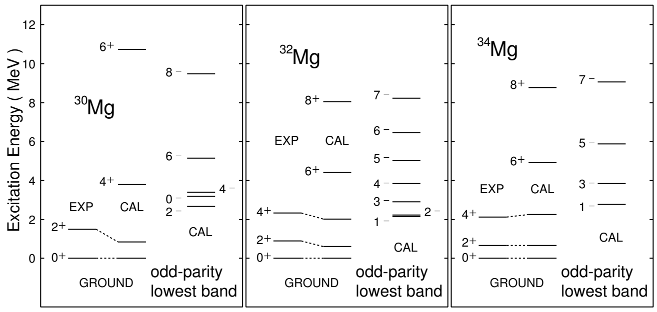


**Fig. 2.** Excitation spectra for the lowest positive- and negative-parity bands in $^{30,32,34}$Mg.

Lagrange multipliers to fix the center of mass and to orthonormalize $|\phi_i\rangle$, respectively. We solve eq. (1) using the imaginary-time method. The left-hand-side of eq. (1) plays a role of the gradient of the energy functional. Numerically, the 3D Cartesian coordinate space is discretized into uniform square grid points on which the single-particle wave functions are represented. The grid spacing is taken to be 0.8 fm, and the grid points inside a sphere of radius 8 fm are used in the calculations. After obtaining the self-consistent intrinsic solutions, $|\Phi\rangle$, we make the angular momentum projection (AMP) to calculate rotational spectra. Since the VAP treatment leads to a triaxial solution in general, we perform the full 3D rotation in Euler angles.

## 3 Low-energy spectra in Mg isotopes

We apply the VAPP method to Mg isotopes. The Skyrme functional with the SGII parameter set is used in the calculation. For the stable nucleus, $^{24}$Mg, we obtain a deformed ground state with positive parity. The deformation is $\beta_2 = 0.52$ with some octupole components, $\beta_{30} = 0.17$ and $\beta_{31} = 0.12$. The state has a dominant $K^\pi = 0^+$ character. The angular momentum projection reproduces experimental spectra and $B(E2)$ for the ground band up to $J^\pi = 6^+$. However, the well-known side band is not reproduced in the calculation because of the $K^\pi = 0^+$ character of the intrinsic state. The negative-parity solutions ($K^\pi = 0^-, 3^-, 1^-$) also well correspond to experimental data, though calculated band-head energies are slightly higher than the experiments by 1–2.5 MeV.

Next, let us discuss results for neutron-rich even-even Mg isotopes, $^{30,32,34}$Mg. Calculated density distributions for the ground state and the lowest negative-parity state are shown in fig. 1. The rotational spectra obtained from these intrinsic states are displayed in fig. 2. Although we show only the lowest solutions in these figures, we have obtained quasi-stationary solutions as well both in the positive- and in the negative-parity sectors. In $^{30,32,34}$Mg, there are two positive-parity solutions, one of which has a large quadrupole deformation of $\beta \approx 0.4$–$0.6$ and the other has a small deformation of $\beta \approx 0.1$–$0.3$. These two states are nearly degenerate in energy, and the interplay between these two seems to be a characteristic feature in the neutron-rich Mg isotopes. In $^{30}$Mg, the one with a small deformation is the ground state, while the one with a large deformation becomes the lowest in $^{32,34}$Mg. The ground state in $^{30}$Mg has a deformation of $\beta_2 = 0.22$ and $\beta_{33} = 0.14$. In $^{32}$Mg, we have the ground state with $\beta_2 = 0.44$ and $\beta_{33} = 0.12$. The rotational correlation is essential to obtain the well-deformed ground state in $^{32}$Mg. The deformation in $^{34}$Mg is even larger, $\beta_2 = 0.53$ with $\beta_{30} = 0.17$. This change of the ground-state character accounts for the experimental observation. The calculated $B(E2; 0^+ \to 2^+)$ are also consistent with the experiments. The moments of inertia are somewhat overestimated in $^{30,32}$Mg, probably because the pairing correlation is neglected in the calculation. The calculation predicts low-energy negative-parity bands in these isotopes. The octupole deformations for the negative-parity states turns out to be smaller than those in the ground states. This is different from what we observed for stable nuclei, $^{20}$Ne and $^{12}$C, in which the octupole deformation is enhanced in negative-parity solutions [1]. The lowest negative-parity excitations in $^{30,32,34}$Mg seem to correspond neither to parity-inversion doublets nor to octupole vibrations.

## 4 Conclusion

We have studied low-energy low-spin spectra for neutron-rich Mg isotopes using the VAPP method. The ground-state deformation becomes larger as the neutron number increases. This is consistent with experimental data. For the lowest negative-parity bands, though there are no experimental information available, the calculation may suggest non-collective excitation character in $^{30,32,34}$Mg.

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